Close Thu: 15.2, 15.3 (integrating!)

Close Tue, May 16: 15.4, 15.5 (finish early)

Exam 2, May 16th (13.3/4,14.1/3/4/7,15.1-15.5)

Office Hours Today: 1:30-3:00pm (Smith 309)

Entry task: Evaluate

$$(a) \int_2^6 \int_1^8 y \, dxdy$$

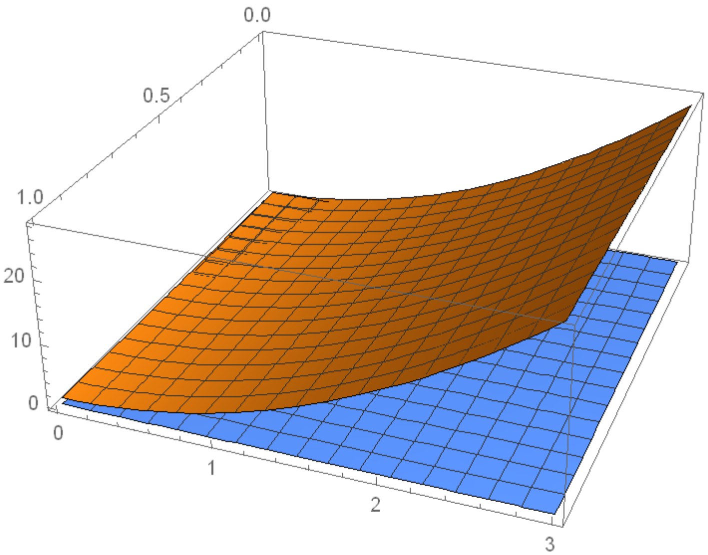
$$(b) \int_2^6 \int_1^8 1 \, dxdy$$

$$Note: \iint_R 1 \, dA = \text{Area of R}$$

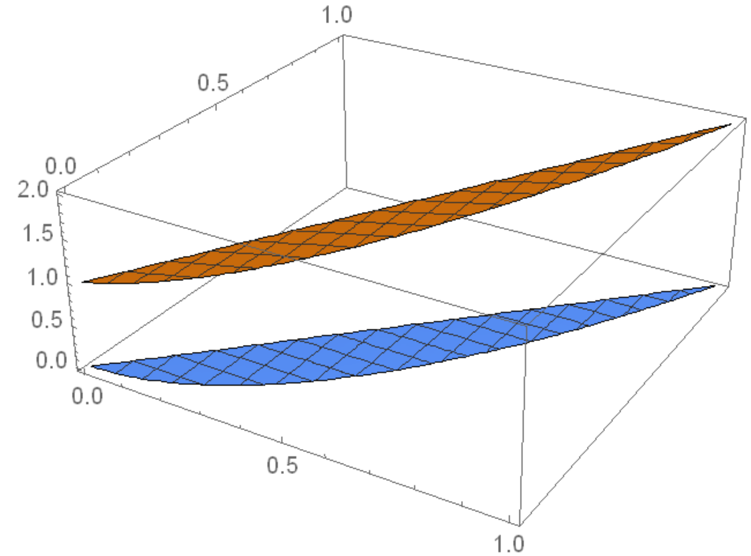
15.3 Double Integrals over General Regions

Type 1 (Top/Bot)	Type 2 (Left/Right)
Given a particular *x* in the range: $a \leq x \leq b,$ we have $g_1(x) \leq y \leq g_2(x)$	Given a particular *y* in the range: $c \leq y \leq d,$ we have $h_1(y) \leq x \leq h_2(y)$
$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dydx$	$\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dx\, dy$

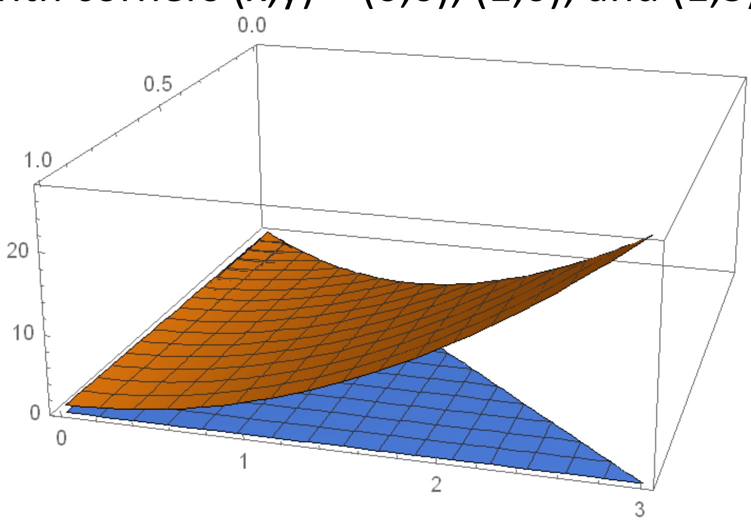
The surface $z = x + 3y^2$ over the rectangular region $R = [0,1] \times [0,3]$



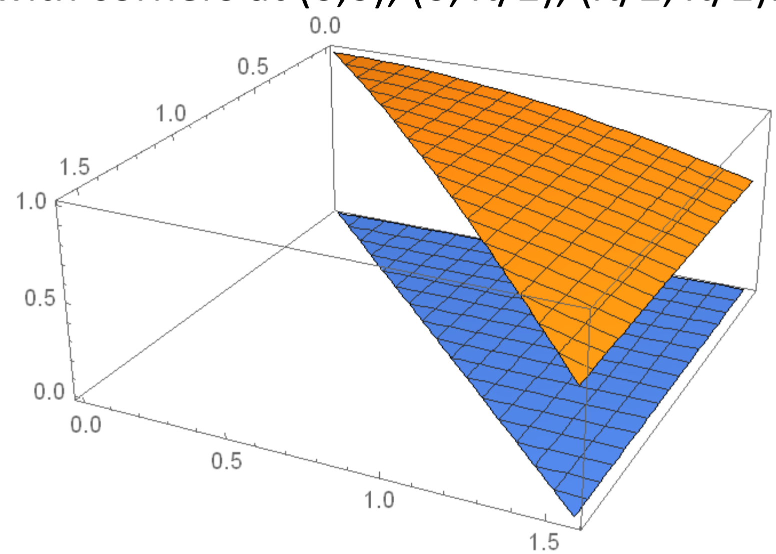
The surface $z = x + 1$ over the region bounded by $y = x$ and $y = x^2$.



The surface $z = x + 3y^2$ over the triangular region with corners $(x,y) = (0,0)$, $(1,0)$, and $(1,3)$.



The surface $z = \sin(y)/y$ over the triangular region with corners at $(0,0)$, $(0, \pi/2)$, $(\pi/2, \pi/2)$.



Examples:

1. Let D be the triangular region in the xy-plane with corners (0,0), (1,0), (1,3).

   Evaluate $\iint_D x + 3y^2 dA$

2. Find the volume of the solid bounded by the surfaces $z = x + 1, y = x^2, y = 2x,$ $z = 0.$

Setting up a problem given in “words”:

1. ***Find integrand***

   Solve for “z” anywhere you see it.
   If there are two z’s, then set up two double integrals (subtract at end).

2. ***Region***

   Graph the region in the *xy*-plane.
   a) Graph all given *x* and *y* constraints.
   b) And find the xy-curves where the surfaces (the z’s) intersect.

Examples (directly from HW):

HW 15.3/10:

Find the volume enclosed by $z = 4x^2 + 4y^2$ and the planes $x = 0$, $y = 2$, $y = x$, and $z = 0$.

HW 15.4/8:

Find the volume below $z = 18 - 2x^2 - 2y^2$ and above the xy-plane.

HW 15.4/9:

Find the volume enclosed by

$-x^2 - y^2 + z^2 = 22$ and $z = 5$.

HW 15.4/10:

Find the volume above the upper cone

$z = \sqrt{x^2 + y^2}$ and below $x^2 + y^2 + z^2 = 81$

Reversing Order of Integration

Examples

1. Draw the region of integration for

$$\int_0^{\pi/2}\int_x^{\pi/2} \frac{\sin(y)}{y}\, dy\, dx$$

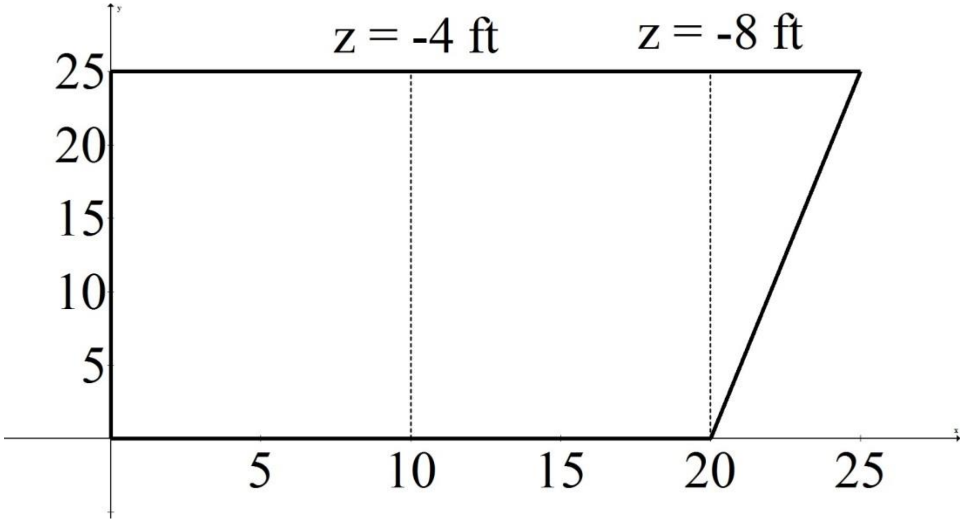
then switch the order of integration.

2. Switch the order of integration for

$$\int_0^{4}\int_{\sqrt{x}}^{2} \sin(y^3)\, dydx$$

An applied problem:

Your swimming pool has the following shape (viewed from above)



The bottom of the pool is a plane with depths as indicated (the pool gets deeper in a linear way from left-to-right)

Solution:

1. Describe the surface (what is *z*?):
   Slope in y-direction = 0
   Slope in x-direction = -4/10 = -0.4
   Also the plane goes through (0, 0, 0)
   Thus, the plane that describes the bottom of the pool is: **z = -0.4x + 0y**

2. Describe the region in *xy*-plane:
   The line on the right goes through (20,0) and (25,25), so it has slope = 5 and it is given by the equation
   y = 5(x-20) = 5x – 100
   or **x = (y+100)/5 = 1/5 y + 20**
   The best way to describe this region is by thinking of it as a left-right region.
   On the left, we always have x = 0
   On the right, we always have x = 1/5 y + 20

Therefore, we have

$$\int_0^{25}\left(\int_0^{\frac{1}{5}y+20} -0.4\,x\,dx\right)dy = -741.\overline{6}\ \ \text{ft}^3$$

15.4 Double Integrals over Polar Regions

Recall:

θ = angle measured from positive x-axis

r = distance from origin

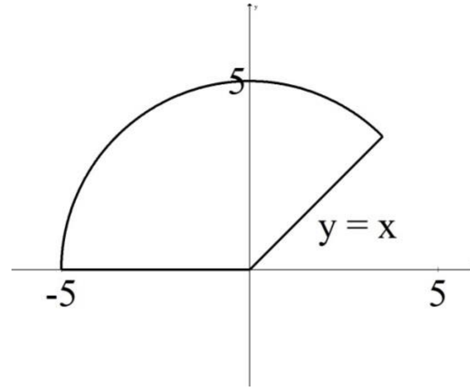
$x = r\cos(\theta)$, $y = r\sin(\theta)$, $x^2 + y^2 = r^2$

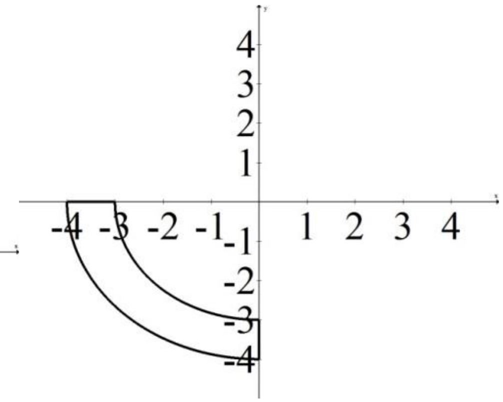
To set up a double integral in polar we will:

1. Describing the region in polar
2. Replace "x" by "r cos(θ)"
3. Replace "y" by "r sin(θ)"
4. Replace "dA" by "r dr dθ"

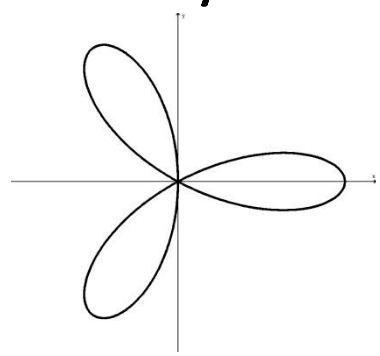
Step 1: Describing regions in polar.

Examples: Describe the regions

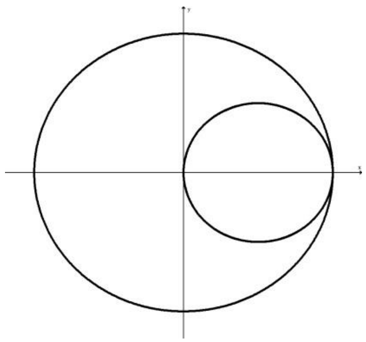




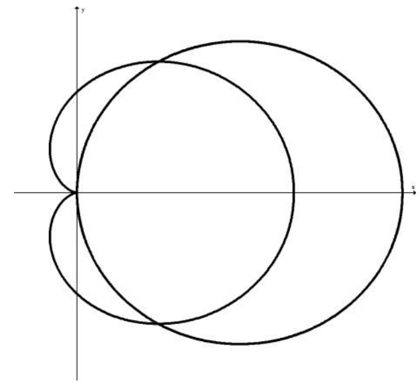
HW 15.4/5: One loop of $r = 6\cos(3\theta)$.



HW 15.4/4: Region in the first quadrant between the circles $x^2 + y^2 = 16$ and $x^2 + y^2 = 4x$.



HW 15.4/7:
Describe the region inside $r = 1+\cos(\theta)$ and outside $r = 3\cos(\theta)$.



Step 2: Set up your integral in polar.

Conceptual notes:

Cartesian

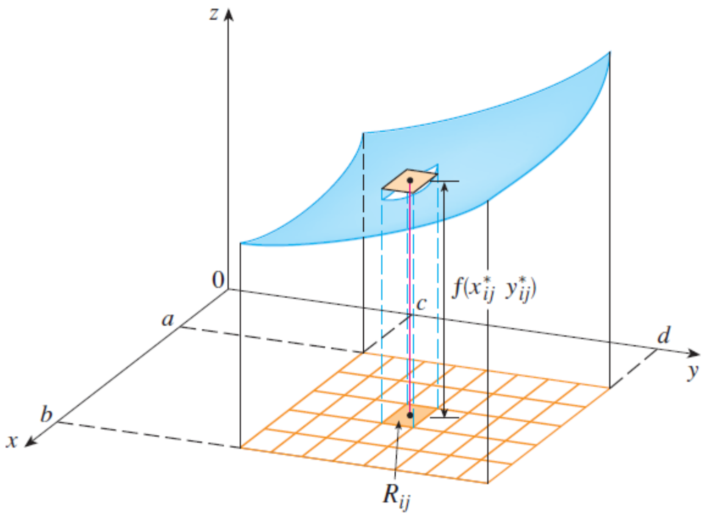
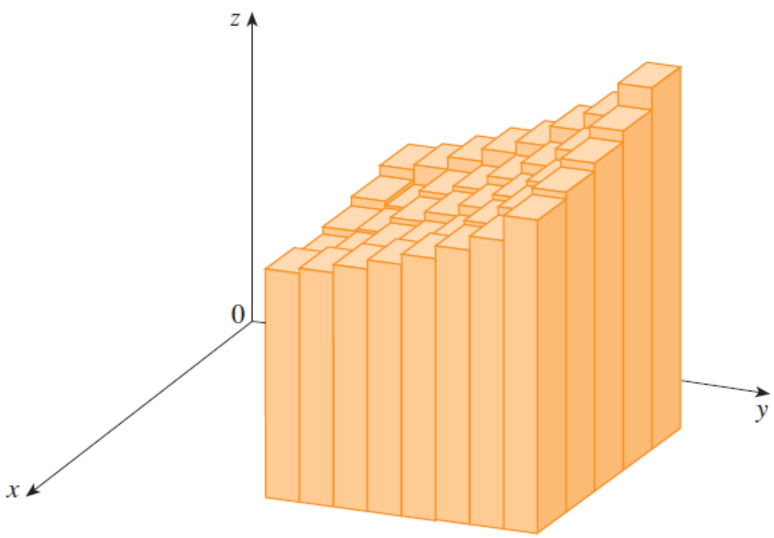
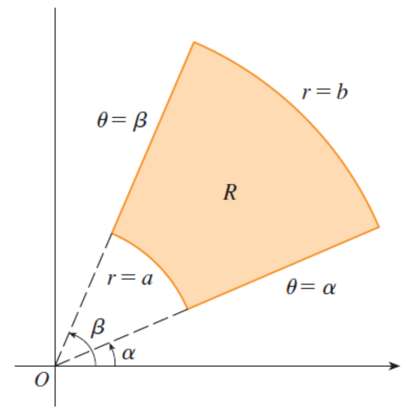
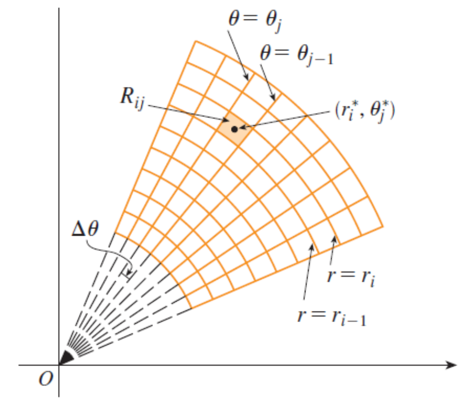


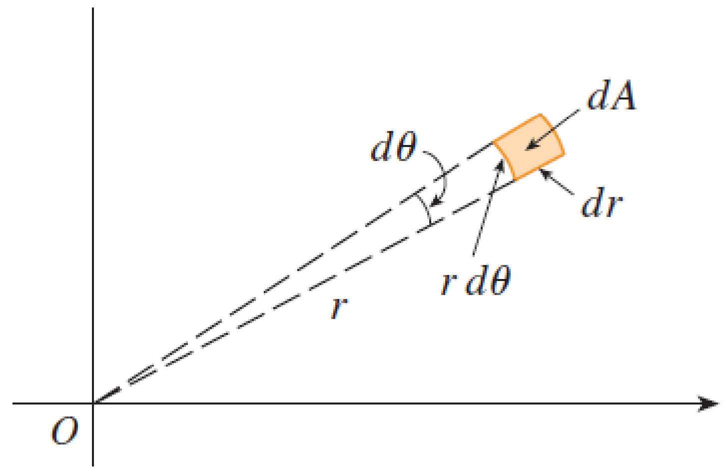
FIGURE 4



Polar







Examples:

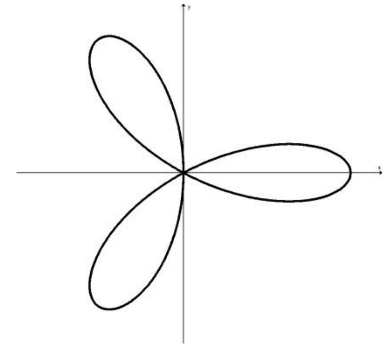
1. Compute

$$\iint_R \frac{\cos(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}\,dA$$

where R is the region in the first quadrant that is between $x^2 + y^2 = 49$, $x^2 + y^2 = 25$ and below y = x.

2. **HW 15.4/5**:

Find the area of one closed loop of r = 6cos(3θ).



3. **HW 15.4/4**:
Evaluate

$$\iint_R x \, dA$$

over the region in the first quadrant between the circles $x^2 + y^2 = 16$ and $x^2 + y^2 = 4x$ using polar

Moral:

Three ways to set up a double integral:

"Top/Bottom":

$$\iint_R f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

"Left/Right":

$$\iint_R f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

"Inside/Outside":

$$\iint_R f(x,y)dA = \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos(\theta), r\sin(\theta))\, r\,dr\,d\theta$$